Math 150 - Foundations: Mathematical Reasoning
Pool Table Investigation - Part I
Spring 2011

Investigating a mathematical question is very similar to a scientific investigation. It usually involves doing many examples (collecting data/making observations), making conjectures about why the observed patterns or results are occurring (making hypotheses) trying to verify the conjecture (conducting an experiment to test the hypothesis) and then, if necessary, going through the process again (and perhaps several times) to refine the conjectures and then, for mathematicians, prove the conjecture is always true. In this activity you will do such a mathematical investigation.

The set up for the question you will be investigating is as follows: Suppose we playing pool on a rectangular pool table and we shoot a ball from the lower left-hand corner at a perfect 45° angle. We will assume the ball will continue to travel along this path and bouncing off the walls at 45° angles until it ends up in a corner. Two possible paths are shown below; the first one is on a 3×4 table and the second on a 4×10 table. On both tables the ball's path begins in the lower left corner.

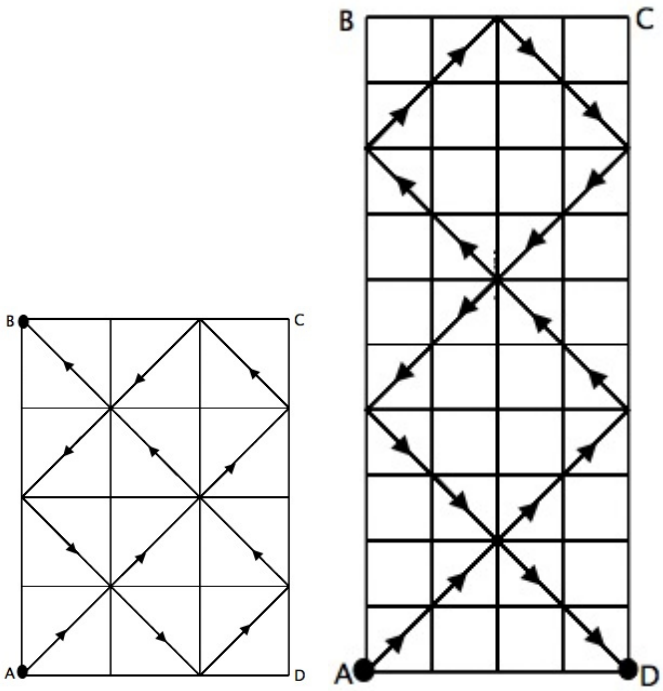

FIGURE 1. Two Possible Paths

The question you are to investigate is how do the dimensions of the pool table determine the corner at which the ball ends? To simplify this, always begin in the lower left corner, label the beginning corner A and then label the other corners B, C, and D in a clockwise direction (as in the tables shown in Figure 2); in addition, when drawing tables whose dimensions are unequal let us agree that the longer dimension will always be the vertical side (again, as in the tables shown in Figure 2).

Answer the following questions completely.

1. Using the graph paper provided, draw at least 10 pool tables of varying widths and lengths. Using the examples above as guides, determine the path the ball will take on each table you have drawn and in which corner, A, B, C or D the ball will exit.
2. For each of the tables 1 - 3 from Question (1) multiply the length and width by 2, draw the new larger table with these dimensions and then determine the path the ball will take on each table you have drawn and in which corner, A, B, C or D the ball will exit.

3. For each of the tables 4 - 6 from Question (1) multiply the length and width by 3, draw the new larger table with these dimensions and then determine the path the ball will take on each table you have drawn and in which corner, A, B, C or D the ball will exit.
4. For each the tables 7 - 8 from Question (1) multiply the length and width by 4, draw the new larger table with these dimensions and then determine the path the ball will take on each table you have drawn and in which corner, A, B, C or D the ball will exit.
5. For each the tables 9 - 10 from Question (1) multiply the length and width by 5, draw the new larger table with these dimensions and then determine the path the ball will take on each table you have drawn and in which corner, A, B, C or D the ball will exit.
6. Based on your answers to Questions (1) - (5) What effect does scaling the length and width by the same number have on the path of the ball? Explain.
7. Now group the 20 or so tables from Questions (1) - (5) by the corner in which the ball exits. While these all have different dimensions, are there commonalities for either the lengths or the widths that most of the tables have? Explain.
8. For those tables that do not fit the commonalities described in Question (7), your answer to Question (6) should suggest an arithmetic operation you could do to the dimensions so that the answers to fit in with the commonalities. Explain.
9. Use your answers Questions (7) and (8) to completely describe a way to determine which corner the ball will exit on any pool table without having to draw the path. Notice that we always start from the lower left corner. Formulate a statement that answers this question, and write a paragraph or two (or more) that completely supports your statement.
10. Below are some giant pool tables that **are not** drawn to scale. Use your answer to Question (9) to indicate, for each table, in which corner the ball will end up.

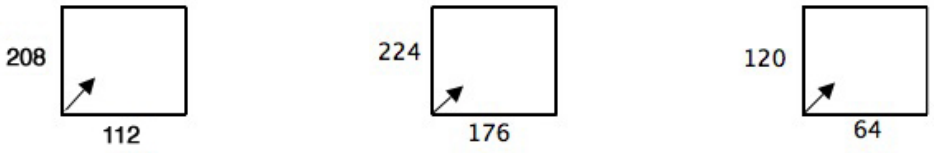

FIGURE 2. Three Large Tables

11. What sizes of pool table will have a path that passes through the middle of every square? What sizes of pool table will have a path that passes through the middle of only some of the squares? If you need to, experiment until you have a guess, and then formulate a statement that answers these two questions. Then, write a paragraph or two (or more) that completely supports your statement.
12. How many times will the ball bounce off of a wall during its path? Formulate a statement that answers this question, and write a paragraph or two (or more)that completely supports your statement.
13. How are patterns integral to determining answers to the questions above?