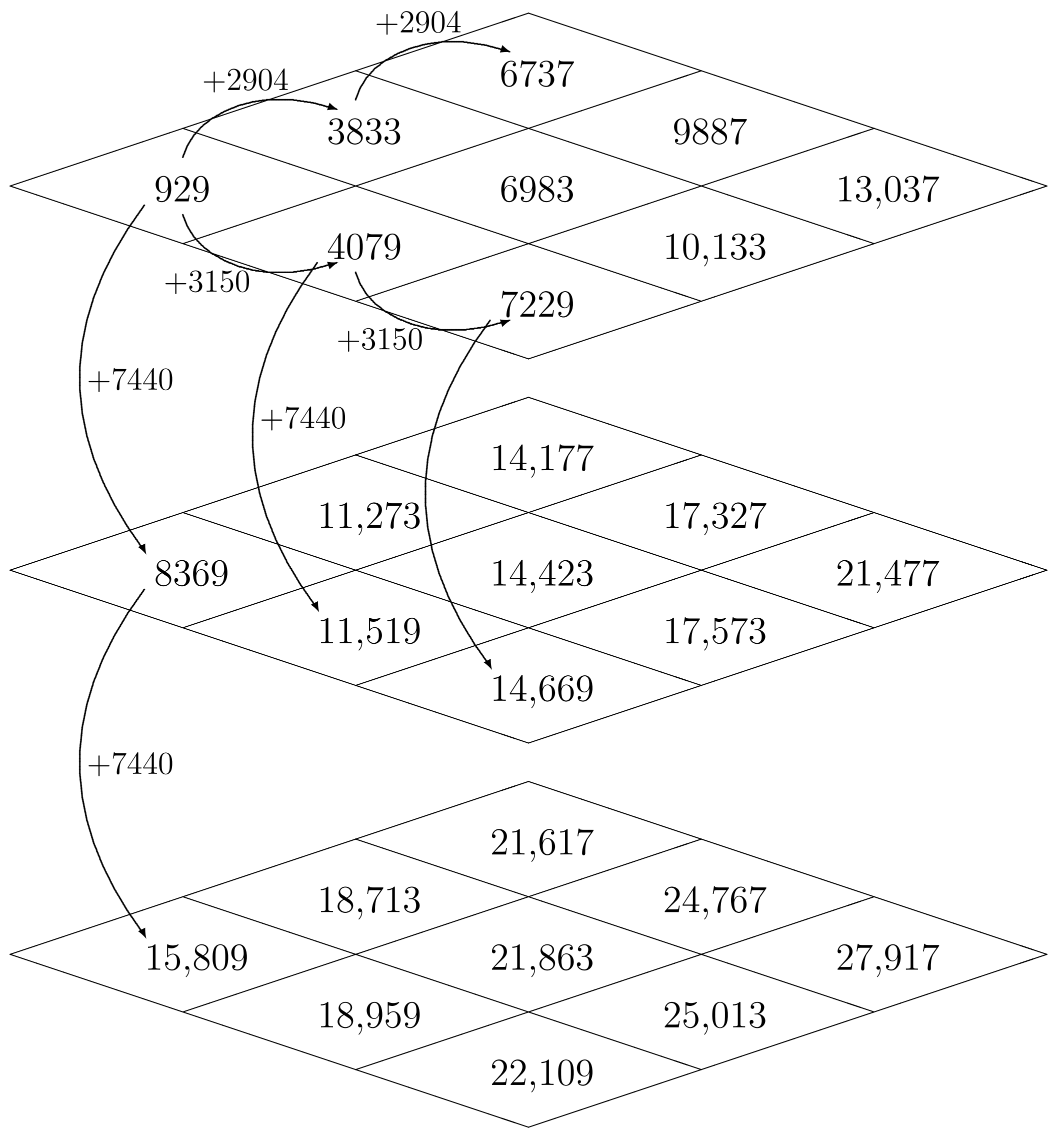
Diagram of a Generalized Arithmetic Progression

This is the smallest possible $3 \times 3 \times 3$ example



A number is *prime* if it has no divisors except 1 and itself. So, for example, 3,5,7 are all prime numbers. A number is *composite* if it is not prime, so, for example, 6 is composite because it is divisible by 2 and also by 3.

An ancient Greek mathematician named Euclid (better known for his work in geometry) showed that there are infinitely many primes – no matter how far out you go in the whole numbers, there are more prime numbers after that point.

An *arithmetic progression* is a sequence of numbers where, to get from one entry to the next, you add a specified number. For example:

$$3, 10, 17, 24, 31, 38, 45, 52, 59, 66, \ldots$$

is an arithmetic progression with starting value 3, where you add 7 to get from one entry to the next.

If you look at this arithmetic progression, you notice that the first number is prime, but the second one is composite. If you think about it for a moment, you will see that if the amount that you add to get from one step to the next is odd, you will never get two prime numbers in a row, because you will alternate from even numbers to odd numbers (even numbers are always divisible by 2, so they are all composite except for 2 itself). This type of thinking allowed us to narrow down the possible examples that we had to look at.

So we can ask: Can we construct examples of arithmetic progressions where all of the first 3 numbers are prime? The answer is not terribly hard: Yes. 3,5,7 is one example; 11,29,47 is another example.

We extend this problem in two ways: Build longer arithmetic progressions that start with more primes in a row, or look at *generalized arithmetic progressions*.

A generalized arithmetic progression is one where we add a specified amount in more than one direction; for example:

$$\begin{matrix} 3 & 7 \\ 19 & 23 \end{matrix}$$

When we move downward, we always add 16; when we move across, we always add 4. In this case, we are showing a 2×2 generalized arithmetic progression of primes (since all 4 of the numbers are prime).

We found the first ever example of a $3 \times 3 \times 3$ generalized arithmetic progression, and then soon found several more.

The example that is in the diagram is the smallest example of a $3 \times 3 \times 3$ generalized arithmetic progression having only prime numbers. Looking at the diagram: anytime we move on the same layer right and up, we add 2904; anytime we move on the same layer right and down, we add 3150; anytime we move from one layer to the one below, we add 7440.

We were able to prove that it is impossible to find a another example where the largest prime number in that example is any smaller than 27,917, so this is the smallest possible example in that sense.