

Gas Laws and Kinetic Molecular Theory

The Gas Laws are based on experiments, and they describe how a gas behaves under certain conditions. However, Gas Laws do not attempt to explain the behavior of gases. A theory must be developed to explain the behavior of gases.

The Kinetic Molecular Theory is a model that explains some of the behavior of gases. The Kinetic Molecular Theory relies on four assumptions that are shown to be reasonable through experimentation.

- (1) The size of a gas particle is negligible as compared to the volume of the container in which the gas is placed.

Gases are mostly empty space, and this is evident because gases can be easily compressed. It is easy to reduce the volume of a gas as compared to reducing the volume of a solid.

- (2) Gases are in rapid motion, and they undergo elastic collisions with each other and the walls of the container; that is, momentum and energy is transferred not lost during collisions.

Gases expand spontaneously to fill any container (rapid motion). Brownian motion of smoke particles (rapid motion). In an insulated container gases do not slow down and eventually condense (elastic collisions).

- (3) The gas molecule do not interact with each other except for colliding with each other.

Gases expand to completely fill a container; they would not if they were attracted to each other.

We will not derive the KMT.

For a single particle the pressure times volume in a cube with sides of length L is

$$PV = \frac{1}{3} mu^2$$

Divide each side by volume.

$$P = \frac{1}{3V} mu^2$$

$$P = \frac{1}{3L^3} mu^2$$

Pressure is just force per unit area, and this equation below demonstrates that the pressure exerted by a gas particle is determined by the force with which the particle collides with the container, divided by the surface area of the container.

$$P = \frac{1}{3L^2} \frac{mu^2}{L}$$

area
Force

$$P = \frac{1}{6L^2} \frac{2 mu^2}{L}$$

area for 6 sides of cube
analyze units

$$\frac{kg \cdot m^2}{m \cdot s^2} \text{ which is } \frac{kg \cdot m^2}{m \cdot s^2}$$

But we need to know the pressure exerted by all the molecules in the container.

Because the velocity of all the gas particles is different, we will have to average the velocity² of the gas particles and then multiply by the total number of gas particles present

$$PV = \frac{1}{3} m \frac{(u_1^2 + u_2^2 + u_3^2 + u_n^2)}{N} N$$

↑
↑
 average of the squared velocities number of particles

$$PV = \frac{1}{3} N m \overline{u^2}$$

remember N is just the number of particles

Experimentation shows that $PR = nRT$. Substitute nRT for PV in the equation above.

$$nRT = \frac{1}{3} N m \overline{u^2}$$

Hey, something cool here...in a geeky sort of way.

$$nRT = \frac{1}{3} N m \overline{u^2}$$

T is proportional to $\overline{u^2}$.

That is, molecular motion increases with increasing temperature.

Using the equation above it is possible to relate the temperature of an ideal gas to the average kinetic energy of a gas particle.

Since,

$$\overline{KE} = \frac{1}{2} m \overline{u^2}$$

$$nRT = \frac{1}{3} N m \overline{u^2}$$

and multiply each side of

by 3 and divide each side by 2.

$$\frac{3}{2} nRT = N \frac{1}{2} m \overline{u^2}$$

$$\frac{3}{2} \frac{RT}{N_A} = \overline{KE}$$

Since the number of particles can be expressed as number of moles times Avagadro's number, $N = n N_A$.

$$\frac{3}{2} \frac{nRT}{n N_A} = \overline{KE}$$

$$\frac{3}{2} \frac{RT}{N_A} = \overline{KE}$$

Comparing KMT to $PV = nRT$

$$PV = \frac{1}{3} N m \overline{u^2}$$

At constant P and T (therefore u), V increases with an increase of number of gas particles "N" due to an increase in the number of collisions.

A decrease in V at constant T (and therefore u) and N results in an increase in pressure due to an increase in the frequency of collisions, and a decrease in the surface area over which the force of each collision is dispersed.

Increasing the Temperature increases the speed the molecules are traveling and this increases the force exerted by the collisions (because each collision involves a larger change in momentum).

To keep the pressure constant the volume must increase so that (1) there are fewer collisions per unit time, and (2) the force of each collision is distributed over a larger area.

At constant volume an increase in the speed of the molecules results in an increase in pressure because (1) as the speed of the particles increases each impact exerts more force on the container, and (2) there are more impacts per unit time.

$$PV = nRT$$

Avagadro's hypothesis: at constant P and T,

$$V = kn$$

Boyle's Law: at constant composition (n) and T,

$$PV = k$$

Charles's Law: at constant composition (n) and pressure

$$V = kT$$

No one's Law: at constant composition (n) and volume

$$P = kT$$

ther things that come out of KMT

$$PV = \frac{1}{3} N m \overline{u^2}$$

$$nRT = \frac{1}{3} N m \overline{u^2}$$

n is number of moles, and N is number of particles

The relationship between the velocities of two samples of gas containing the same number of particles at the same temperature, pressure, and volume.

KMT tells us that if the pressures of two gases are the same then the force one gas exerts against the wall of its container must be equal to the force that the other gas exerts against the wall of its container. That is, the total force exerted by all the particles must be the same in the heavy gas and the light gas. Since the number of particles is the same the lighter gas must be moving faster than the heavier gas even though the temperatures of the gases are the same.

he relationship between mass and speed can be demonstrated mathematically as the root mean squared velocity of a gas.

$$\overline{u^2} = \frac{3 RT}{N_A m}$$

$$\sqrt{\overline{u^2}} = \sqrt{\frac{3 RT}{N_A m}}$$

$$u_{\text{RMS}} = \sqrt{\frac{3 RT}{N_A m}}$$

Root mean squared (RMS) velocity is not the average velocity, but it is as close as we can get to the average velocity with out knowing the distribution of the velocities of the particles. However, at low temperatures the RMS velocity is close to the average velocity.

Since the distribution of the velocities is a Boltzman distribution it is possible to calculate the average velocity, but this is beyond the scope of this class.